Profesor:
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PITAGORAS ACADEMIA

ÁLGEBRA

GRUPO PITÁGORAS

INECUACIONES IRRACIONALES

1.- FORMA:

Las inecuaciones irracionales pueden ser de la forma:

$$\sqrt[m]{P_{(x)}} \gtreqless Q_{(x)} \qquad \sqrt[m]{P_{(x)}} \lesseqgtr \sqrt[n]{Q_{(x)}}$$

2.- CONSIDERACIONES PREVIAS A LA SOLUCIÓN

$1.-\sqrt[2n]{P(x)} \in \mathbb{R} \wedge n \in \mathbb{Z}^{+} \quad \rightarrow \quad P(x) \geq 0$

$2.-\dfrac{1}{\sqrt[2n]{P(x)}} \in \mathbb{R} \wedge n \in \mathbb{Z}^{+} \quad \rightarrow \quad P(x) > 0$

$3.-\sqrt[2n]{P(x)} \in \mathbb{R} \wedge n \in \mathbb{Z}^{+} \quad \rightarrow \sqrt[2n]{P(x)} \geq 0$

$4.-\dfrac{1}{\sqrt[2n]{P(x)}} \in \mathbb{R} \wedge n \in \mathbb{Z}^{+} \quad \rightarrow \dfrac{1}{\sqrt[2n]{P(x)}} > 0$

$Sean\ a, b \in \mathbb{R}\ entonces:$

$5.- \quad a \geq b \geq 0 \quad entonces \quad a^2 \geq b^2 \geq 0$

$6.- \quad a \geq b \geq 0 \quad entonces \quad a^3 \geq b^3 \geq 0$

$7.- \quad En\ general \quad a \geq b \geq 0 \quad entonces \quad a^n \geq b^n \geq 0$

$8.- \quad a > b \geq 0 \quad entonces \quad a^2 > b^2 \geq 0$

$9.- \quad a > b \geq 0 \quad entonces \quad a^3 > b^3 \geq 0$

$10.- \quad En\ general \quad a > b \geq 0 \quad entonces \quad a^n > b^n \geq 0$

$11.- \quad a \geq b \quad entonces \quad a^3 \geq b^3$

$12.- \quad En\ general \quad a \geq b \wedge n \in \mathbb{Z}^+ entonces \quad a^{2n+1} \geq b^{2n+1}$

$13.- \quad a > b \quad entonces \quad a^3 > b^3$

$14.- \quad En\ general \quad a > b \wedge n \in \mathbb{Z}^+ entonces \quad a^{2n+1} > b^{2n+1}$

EJERCICIOS NIVEL I

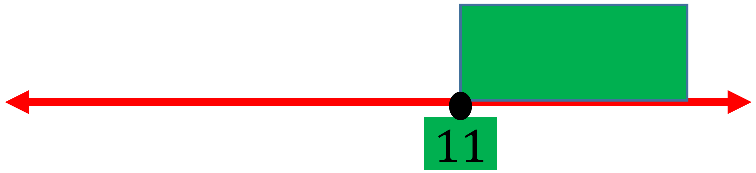
$1.-\ Resolver:$

$$\sqrt{x-2} \geq 3$$

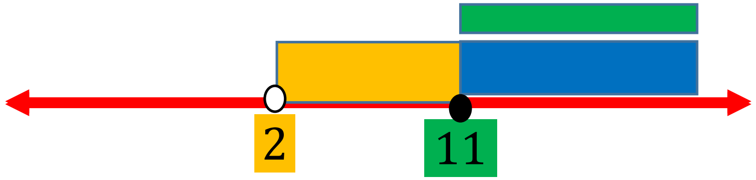
SOLUCIÓN

1) $\sqrt{x-2} \geq 3 > 0 \in \mathbb{R} \quad \rightarrow x-2 > 0 \rightarrow x > 2$

2) $\sqrt{x-2} \geq 3 > 0 \quad \rightarrow x-2 \geq 9 \rightarrow x \geq 11$



3) $Ya\ que\ tanto\ 1)\ como\ 2)\ deben\ ocurrir\ simultáneamente, se\ intersectará\ ambos\ conjuntos$



$\boldsymbol{C.S. = [11; +\infty[}$

EJERCICIOS BÁSICOS

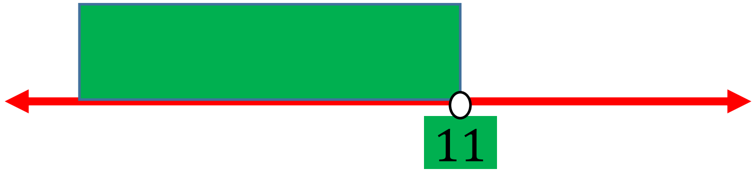
$2. - Resolver:$

$$\sqrt{x+4} < 3$$

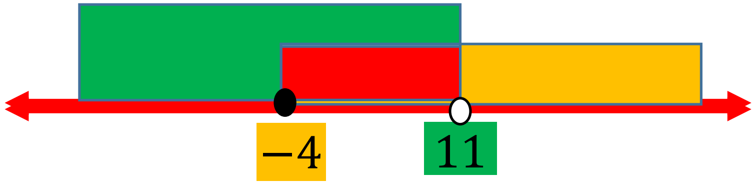
SOLUCIÓN

1) $\sqrt{x+4} \in \mathbb{R} \rightarrow x+4 \geq 0 \rightarrow x \geq -4$

2) $\sqrt{x-2} < 3 \rightarrow x-2 < 9 \rightarrow x < 11$



3) $Ya\ que\ tanto\ 1)\ como\ 2)\ deben\ ocurrir\ simultáneamente, se\ intersectará\ ambos\ conjuntos$



$$\boldsymbol{C.S. = [-4; 11[}$$

EJERCICIOS BÁSICOS

$3.- \ Resolver:$

$$\sqrt{x+1} \geq -3$$

SOLUCIÓN

1) $\sqrt{x+1} \in \mathbb{R} \quad \rightarrow x+1 \geq 0 \rightarrow x \geq -1$

2) $Ya\ que\ \sqrt{x+1} \geq 0\ entonces$

$es\ evidente\ que\ cumple\ \ \sqrt{x+1} \geq -3$

$$\boldsymbol{C.S. = [-1; +\infty[}$$

$4.- \ Resolver:$

$$\sqrt{x+1} \leq -2$$

SOLUCIÓN

1) $Ya\ que\ \sqrt{x+1} \geq 0\ entonces\ es\ imposible\ \sqrt{x+1} \leq -2$

$$\boldsymbol{C.S = \emptyset}$$

EJERCICIOS NIVEL II

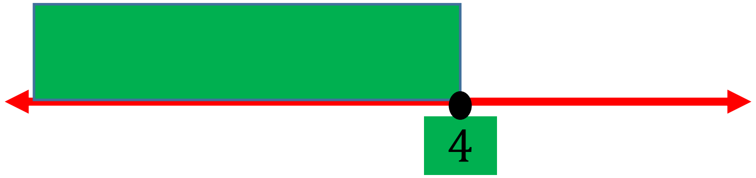
$1. - Resolver:$

$$\sqrt{x-2} \leq 4 - \mathrm{x}$$

SOLUCIÓN

1) $\sqrt{x-2} \in \mathbb{R} \quad \rightarrow x-2 \geq 0 \rightarrow x \geq 2$

2) $0 \leq \sqrt{x-2} \leq 4-x \qquad \rightarrow 0 \leq 4-x \qquad \rightarrow x \leq 4$

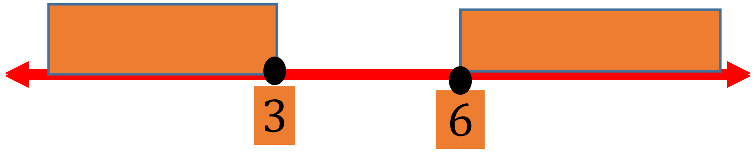


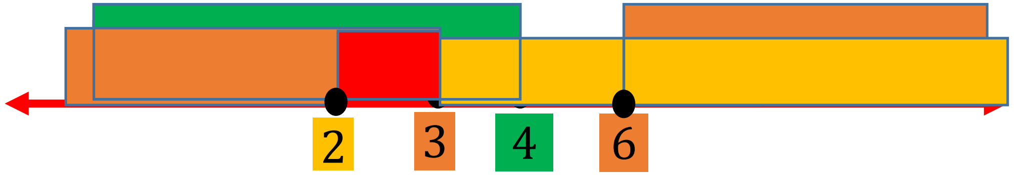
3) $0 \leq \sqrt{x-2} \leq 4-x \qquad \rightarrow \qquad \left(\sqrt{x-2}\right)^2 \leq (4-x)^2$

$$\rightarrow \quad x-2 \leq 16-8x+x^2$$

$$\rightarrow \quad 0 \leq x^2-9x+18$$

$$\rightarrow \quad 0 \leq (x-3)(x-6)$$





$$C.S. = [2, 3]$$

EJERCICIOS NIVEL II

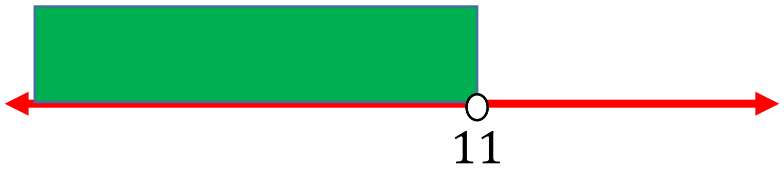
$2. - Resolver:$

$$\sqrt{x-5} < 11 - \mathrm{x}$$

SOLUCIÓN

1) $\sqrt{x-5} \in \mathbb{R} \rightarrow x - 5 \geq 0 \rightarrow x \geq 5$

2) $0 \leq \sqrt{x-5} < 11 - x \quad \rightarrow 0 < 11 - x \quad \rightarrow x < 11$

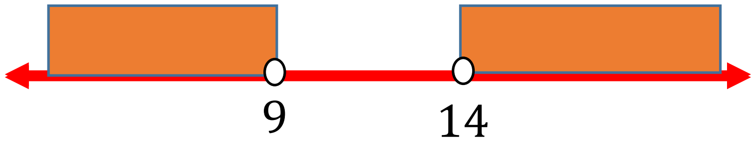


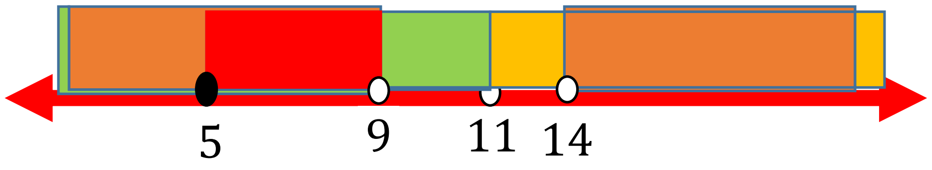
3) $0 \leq \sqrt{x-5} < 11 - x \quad \rightarrow \quad \left(\sqrt{x-5}\right)^2 < (11-x)^2$

$\rightarrow \quad x - 5 < 121 - 22x + x^2$

$\rightarrow \quad 0 < x^2 - 23x + 126$

$\rightarrow \quad 0 < (x-14)(x-9)$



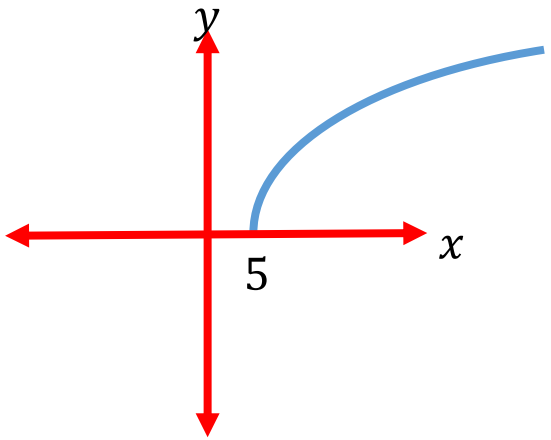


$$\boldsymbol{C.S. = [5, 9[}$$

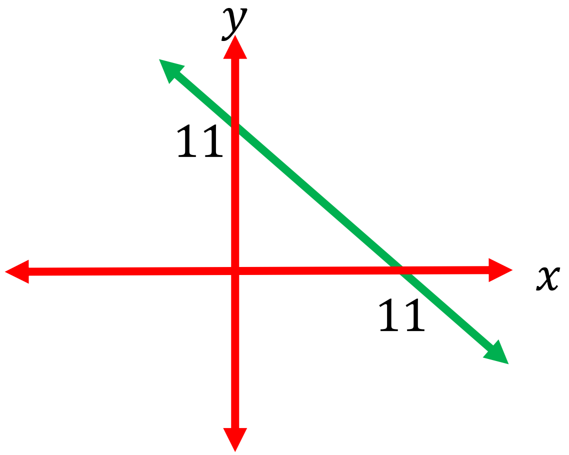
$2.-$ *Resolver*:

$$\sqrt{x-5} < 11 - x$$

SOLUCIÓN2 *Graficando la función:* $f(x) = \sqrt{x-5}$



Graficando la función: $g(x) = 11 - x$



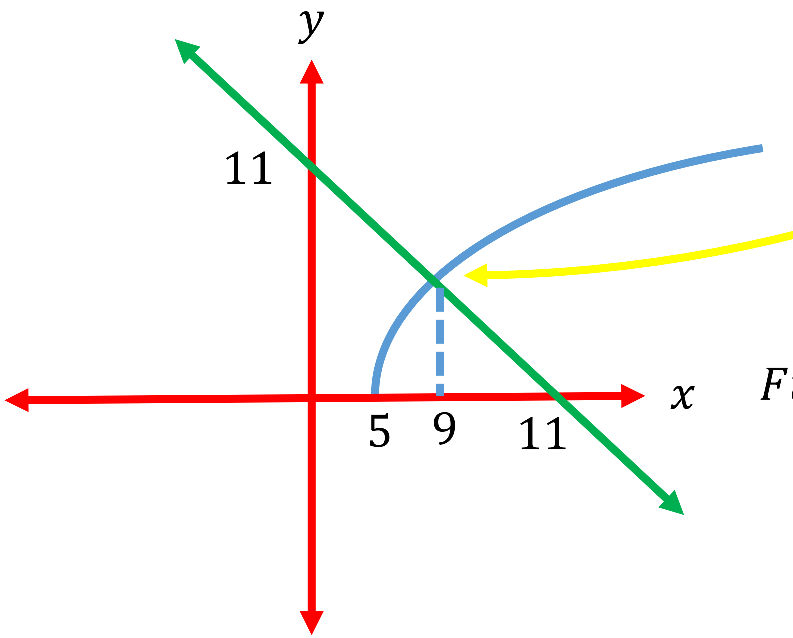
$$\sqrt{x-5} = 11 - x$$

$$x - 5 = 121 - 22x + x^2$$

$$0 = x^2 - 23x + 126$$

$$0 = (x-14)(x-9)$$

$$x = 14 \vee x = 9$$



Finalmente el intervalo en el que

$$\sqrt{x-5} < 11 - x$$

sería $[5; 9[$

EJERCICIOS NIVEL II

$2.- Resolver:$

$$\sqrt{x-3} > 9 - x$$

SOLUCIÓN

1) $\sqrt{x-3} \in \mathbb{R} \;\; \rightarrow x - 3 \geq 0 \rightarrow x \geq 3$

$Ya\ que\ desconocemos\ el\ signo\ de\ 9 - x, analizamos:$

$Sea\ x \epsilon\ [3; 9]$ $\;\; es\ decir: \quad 3 \leq x \leq 9$

$$-9 \leq -x \leq -3$$

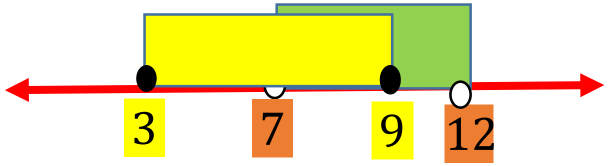
$$0 \leq 9 - x \leq 6$$

$es\ decir: \quad \sqrt{x-3} > 9 - x \geq 0$

$$x - 3 > 81 - 18x + x^2$$

$$0 > x^2 - 19x + 84$$

$$0 > (x-7)(x-12)$$



$de\ donde\ x \;\in]7; 9]$

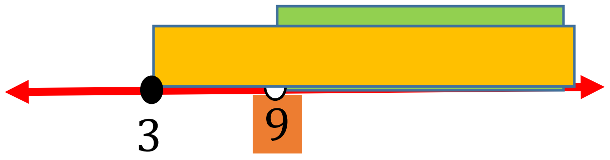
$Sea\ x \epsilon\ ]9; +\infty]$ $\;\; es\ decir: \quad x > 9$

$$-x < -9$$

$$9 - x < 0$$

$pero\ ya\ que \quad \sqrt{x-3} \geq 0 \quad y \quad 9 - x < 0$

$es\ evidente\ que \quad \sqrt{x-3} > 9 - x$



$de\ donde\ x \;\in]9; +\infty]$

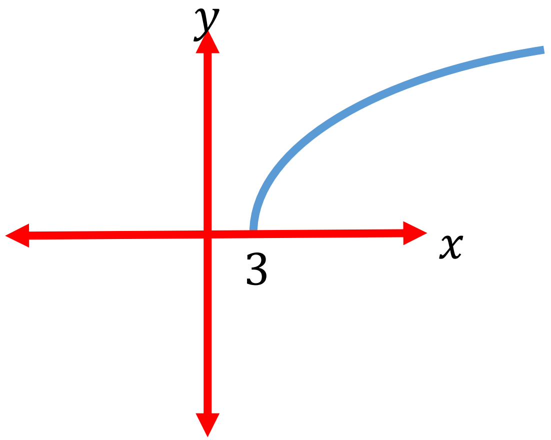
$finalmente\ todas\ las\ soluciones\ serían:$

$$]7; 9] \;\cup\;]9; +\infty] \;=]7; +\infty]$$

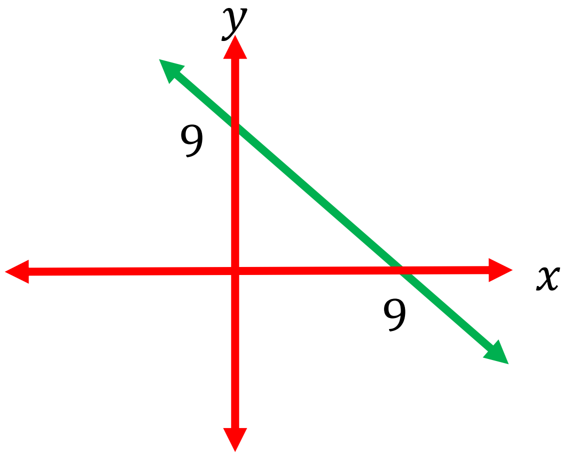
$2.-$ *Resolver*:

$$\sqrt{x-3} < 9 - \mathrm{x}$$

<u>SOLUCIÓN2</u> *Graficando la función*: $\boldsymbol{f(x) = \sqrt{x-3}}$



Graficando la función: $\boldsymbol{g(x) = 9 - x}$



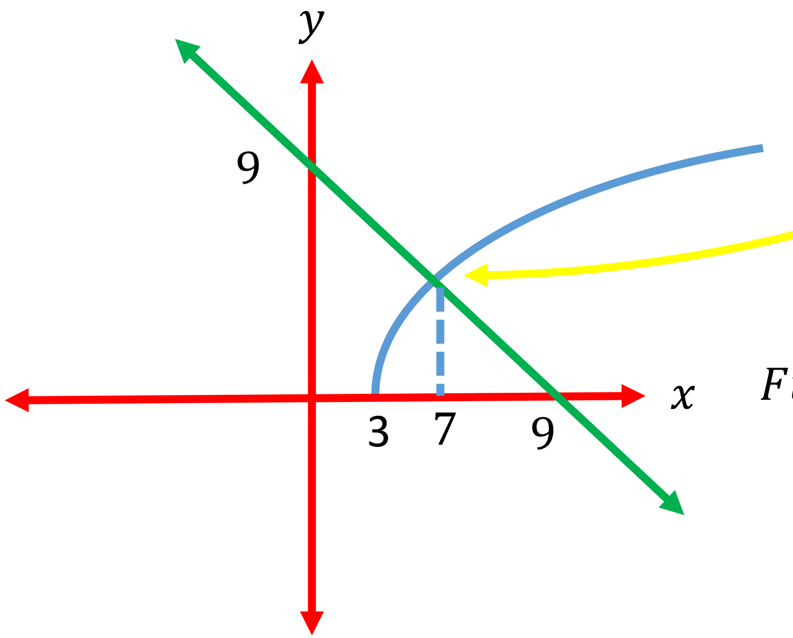
$$\boldsymbol{\sqrt{x-3} = 9 - x}$$

$$x - 3 = 81 - 18x + x^2$$

$$0 = x^2 - 19x + 84$$

$$0 = (x-12)(x-7)$$

$$x = 12 \lor \ x = 7$$



Finalmente el intervalo en el que

$$\boldsymbol{\sqrt{x-3} > 9 - x}$$

sería $\boldsymbol{]7;+\infty[}$

VALOR ABSOLUTO

1.- DEFINICIÓN:

$Sea\ x \ \in \mathbb{R}$

$$|x| = \begin{cases} x & ; x > 0 \\ 0 & ; x = 0 \\ -x & ; x < 0 \end{cases}$$

$Ejemplos:$

$|7| = 7$

$|-2| = -(-2) = 2$

$|\pi - 3| = \pi - 3$

$\left|\sqrt{7} - 4\right| = -\left(\sqrt{7} - 4\right) = 4 - \sqrt{7}$

$Sea\ x \in [4;7[$

$calcular: \quad |x-8| + |x-3|$

$4 \le x < 7$

$-4 \le x - 8 < -1$

$es\ decir\ x - 8 < 0$

$por\ \ lo\ tanto:$

$|x-8| = -(x-8) = 8 - x$

$4 \le x < 7$

$1 \le x - 3 < 4$

$es\ decir\ x - 3 > 0$

$por\ \ lo\ tanto:$

$|x-3| = x - 3$

$por\ lo\ tanto: \quad |x-8| + |x-3| = 8 - x + x - 3 = 5$

2.- PROPIEDADES:

$Se\ cumple\ \ \forall x \in \mathbb{R}$:

$|x| \geq 0$

$|x| = |-x|$

$|x| \geq x$

$|x| \geq -x$

$|x|^2 = x^2 = |x^2|$

$\sqrt{x^2} = |x|$

$Se\ cumple\ \ \forall x, y \in \mathbb{R}$:

$|xy| = |x||y|$

$Se\ cumple\ \ \forall x, y \in \mathbb{R}, \qquad y \neq 0$

$$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$$

$Se\ cumple\ \ \forall x, y \in \mathbb{R}$:

$$\big||x| - |y|\big| \leq |x + y| \leq |x| + |y|$$

$Si\ |x| = y\ \wedge\ y \geq 0\ \ entonces\ \ x = y\ \ \vee\ \ x = -y$

$Si\ |x| < y\ \wedge\ y \geq 0\ \ entonces\ \ -y < x < y$

$Si\ |x| > y\ \wedge\ y \geq 0\ \ entonces\ \ x > y\ \ \vee\ \ x < -y$

$$Si\ \frac{|x| - |y|}{|z| - |w|} \gtreqless 0\ entonces\ \frac{x^2 - y^2}{z^2 - w^2} \gtreqless 0$$

PROBLEMAS RESUELTOS

01

( ) $\forall x \in \mathbb{R}: \; -x-1 \leq |x|$

Demostración:

$\forall x \in \mathbb{R}: \; |x| \geq -x$

$|x| \geq -x-1$ (V)

( ) $\forall x \in \mathbb{R}: \; |x+1| \leq |x|+3$

Demostración: $\forall x \in \mathbb{R}$:

$|x+1| \leq |x| + |1|$

$|x+1| \leq |x|+1 \;\wedge\; |x|+1 \leq |x|+3$

Por transitividad:

$|x+1| \leq |x|+3$ (V)

( ) $\forall x \in \mathbb{R}$:

$-x \leq |x| \rightarrow -|x| \leq x$

$-|x|-3 \leq x$

$x \leq |x| \rightarrow x \leq |x|+2$

$\therefore -|x|-3 \leq x \leq |x|+2$ (V)

$\therefore$ VVV

02

Si $x \in \;]-3; -2[$

$\Rightarrow -3 < x < -2 \quad (-4)$

$-7 < x-4 < -6 \quad (\times 5)$

$-35 < 5x-20 < -30$

$\Rightarrow 5x-20 < 0 \Rightarrow |5x-20| = 20-5x$

$\Rightarrow -3 < x < -2$

$-9 < 3x < -6$

$-29 < 3x-20 < -26$

$\Rightarrow 3x-20 < 0 \Rightarrow |3x-20| = 20-3x$

$\Rightarrow E = \dfrac{|5x-20| - |3x-20|}{x}$

$E = \dfrac{20-5x-(20-3x)}{x}$

$E = \dfrac{-2x}{x} = -2$

Rpta: 2

PITAGORAS ACADEMIA

03 $|x-5| + |y-\sqrt{11}| = \sqrt{-|w+3| - |z-1| + |w+z+2|}$

Ya que: $\sqrt{-|w+3| - |z-1| + |w+z+2|} \geq 0$

$-|w+3| - |z-1| + |w+z+2| \geq 0$

$|w+z+2| \geq |w+3| + |z-1|$

Pero $w+z+2 = (w+3) + (z-1)$

Desigualdad triangular:

$|w+3+z-1| \leq |w+3| + |z-1|$

$|w+z+2| \leq |w+3| + |z-1|$

$|w+z+2| = |w+3| + |z-1|$

$\Rightarrow -|w+3| - |z-1| + |w+z+2| = 0$

$\Rightarrow |x-5| + |y-\sqrt{11}| = 0$

Lo cual solo puede ocurrir si:

$x-5=0 \wedge y-\sqrt{11}=0$

$x=5 \wedge y=\sqrt{11}$

$\Rightarrow$ MÍNIMO VALOR DE

$\sqrt{x^2+y^2+|z|} = \sqrt{5^2+\sqrt{11}^2+|z|} = \sqrt{36+|z|} \rightarrow \text{MIN } z=0$

$\Rightarrow 6$

04

$$|x^2-4| \leq (x+2)^2$$

$$|x+2||x-2| \leq (x+2)^2$$

$$\left(|x+2||x-2|\right)^2 \leq \left((x+2)^2\right)^2$$

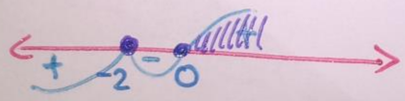
$$(x+2)^2(x-2)^2 \leq (x+2)^4$$

$$(x+2)^2(x-2)^2 - (x+2)^4 \leq 0$$

$$(x+2)^2\left[\underbrace{(x-2)^2-(x+2)^2}\right] \leq 0$$

$$(x+2)^2\left[-4(x)(2)\right] \leq 0$$

$$x(x+2)^2 \geq 0$$



$$x \in [0; \infty^{+}[\cup \{-2\}$$

Ya que $(x+2)^2 \geq 0$

$$\rightarrow \quad |x^2-4| \leq x^2+4x+4$$

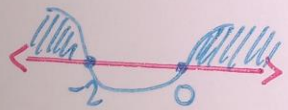
$$-(x^2+4x+4) \leq x^2-4 \leq x^2+4x+4$$

$$-x^2-4x-4 \leq x^2-4 \leq x^2+4x+4$$

$$0 \leq 2x^2+4x \quad \wedge \quad -8 \leq 4x$$

$$0 \leq 2x(x+2) \quad \wedge \quad -2 \leq x$$

$$0 \leq x(x+2) \quad \wedge \quad x \geq -2$$



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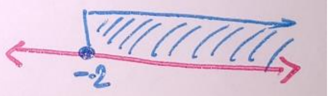
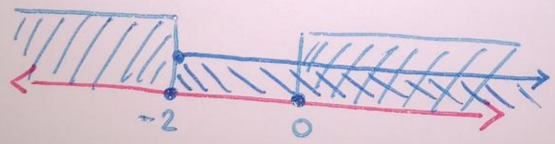




$$x \in [0; \infty^{+}[\cup \{-2\}$$

05

$$\sqrt{(x+1)^2} \leq -x-1$$

$$|x+1| \leq -x-1$$

ya que $|x+1| \geq 0$

$\Rightarrow -x-1 \geq 0 \longrightarrow x+1 \leq 0$

$\longrightarrow x \leq -1$

$\longrightarrow |x+1| = -x-1$

$\Rightarrow$ Reemplazando:

$$-x-1 \leq -x-1$$

lo cual siempre cumplirá

y además se sabe que $x \leq -1$

$\downarrow$

$x \in]-\infty ; -1]$

05

$$\sqrt{(x+1)^2} \leq -x-1$$

$$|x+1| \leq -x-1$$

$$|x+1| + x + 1 \leq 0$$

Sabemos que:

$x > -1 \quad \vee \quad x = -1 \quad \vee \quad x < -1$

$x+1 > 0$

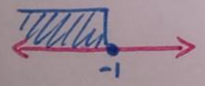
$\Rightarrow |x+1| = x+1$

Reemplazando:

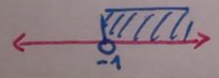
$\Rightarrow x+1+x+1 \leq 0$

$2x+2 \leq 0$

$x \leq -1$



pero $x > -1 \quad \therefore \phi$



$\vee$ Reemplazando:

$|-1+1| + (-1) + 1 \leq 0$

$0 \leq 0$

$\hookrightarrow x = -1$ cumple

$\vee \quad x = -1$

$\vee \quad \{-1\}$

$\vee \quad x+1 < 0$

$\Rightarrow |x+1| = -x-1$

Reemplazando:

$-(x+1) + x + 1 \leq 0$

$0 \leq 0$

$\Rightarrow$ Cumple todo $x < -1$

$]-\infty ; -1[$

$\vee \quad]-\infty ; -1[$

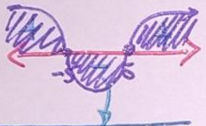
$$x \in \;]-\infty ; -1]$$

(06) $\left|1-|x-8|\right| \geq \sqrt{25-x^2}$

* $\sqrt{25-x^2} \in \mathbb{R} \Rightarrow 25-x^2 \geq 0$

$x^2-25 \leq 0$

$(x+5)(x-5) \leq 0$



$\boxed{-5 \leq x \leq 5}$

* ya que $-5 \leq x \leq 5$

$-13 \leq x-8 \leq -3$

es decir: $x-8<0$

$|x-8| = 8-x$

Reemplazando:

$|1-(8-x)| \geq \sqrt{25-x^2}$

$|x-7| \geq \sqrt{25-x^2}$

* ya que: $-5 \leq x \leq 5$

$-12 \leq x-7 \leq -2$

es decir $x-7<0$

$\Rightarrow |x-7| = 7-x$

Reemplazando:

$7-x \geq \sqrt{25-x^2}$

$(7-x)^2 \geq 25-x^2$

$49-14x+x^2 \geq 25-x^2$

$2x^2-14x+24 \geq 0$

$x^2-7x+12 \geq 0$

$(x-3)(x-4) \geq 0$

pero $x \in [-5;5]$

$\therefore x \in [-5;3] \cup [4;5]$

$\Rightarrow a=-5 \wedge b=3 \wedge c=4 \wedge d=5$

$\Rightarrow |a|+|b|+|c|+|d| = \underline{17}$

(07) $|x+2| < |x-1| + 3$

Sabemos:

$x \geq 1$ ✓

$x-1 \geq 0 \rightarrow |x-1| = x-1$

$x+2 \geq 3 \rightarrow |x+2| = x+2$

Reemplazando:

$x+2 < x-1+3$

$x+2 < x+2$

Lo cual no se cumple $\Rightarrow \varnothing$

$-2 \leq x < 1$ ✓

$-3 \leq x-1 < 0 \rightarrow |x-1| = 1-x$

$0 \leq x+2 < 3 \rightarrow |x+2| = x+2$

Reemplazando:

$x+2 < 1-x+3$

$2x < 2$

$x < 1$

pero $-2 \leq x < 1$

$[-2; -1[$

$x < -2$

$x+2 < 0 \rightarrow |x+2| = -x-2$

$x-1 < -3 \rightarrow |x-1| = 1-x$

Reemplazando:

$-x-2 < 1-x+3$

$-2 < 4$

Lo cual siempre se cumple

$\Rightarrow x \in]-\infty; -2[$

FINALMENTE:

$\underbrace{\varnothing \cup [-2; -1[\cup]-\infty; -2[}$

$]-\infty; -1[$

07) $|x+2| < |x-1| + 3$

Sea $x - 1 = y$

$\hookrightarrow |y+3| < |y| + 3$

$\hookrightarrow \cancel{y^2} + 6y + \cancel{9} < \cancel{y^2} + 6|y| + \cancel{9}$

$y < |y| \rightarrow y < 0$

$x - 1 < 0$

$x < 1$

$\Rightarrow x \in \left]-\infty ; 1\right[$

08

$$\frac{|3x-1|+2x}{|x+1|-3x} \geq 0$$

$x \geq \frac{1}{3}$

$3x \geq 1 \to 3x-1 \geq 0$

$\to |3x-1| = 3x-1$

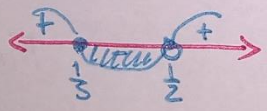
$x+1 \geq \frac{4}{3} \to |x+1| = x+1$

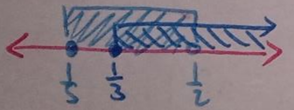
Reemplazando:

$$\frac{3x-1+2x}{x+1-3x} \geq 0$$

$$\frac{5x-1}{-2x+1} \geq 0$$

$$\frac{5x-1}{2x-1} \leq 0$$





$[\frac{1}{3} ; \frac{1}{2}[$

$-1 \leq x < \frac{1}{3}$

$-3 \leq 3x < 1$

$-4 \leq 3x-1 < 0$

$|3x-1| = 1-3x$

$0 \leq x+1 < \frac{4}{3}$

$|x+1| = x+1$

Reemplazando:

$$\frac{1-3x+2x}{x+1-3x} \geq 0$$

$$\frac{-x+1}{-2x+1} \geq 0$$

$$\frac{x-1}{2x-1} \geq 0 \to$$

$[-1 ; \frac{1}{3}[$

$x < -1$

$3x-1 < -4$

$|3x-1| = 1-3x$

$x+1 < 0$

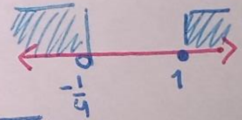
$|x+1| = -x-1$

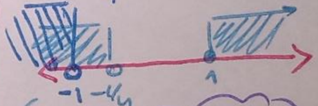
Reemplazando:

$$\frac{1-3x+2x}{-x-1-3x} \geq 0$$

$$\frac{1-x}{-4x-1} \geq 0$$

$$\frac{x-1}{4x+1} \geq 0$$





$]-\infty ; -1[$

UNIÓN: $]-\infty ; \frac{1}{2}[$

09 $$\frac{(x^4+3x^2+10)\left(|x|-1\right)\left(|x|-3\right)}{\left(2|x|-2x+8\right)(x^2-4)} \le 0$$

$\forall x \in \mathbb{R}: \; x^4+3x^2+10>0$

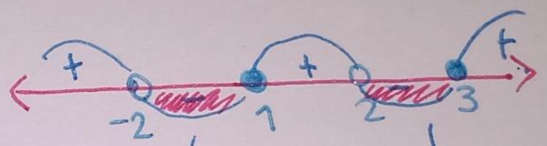
$\forall x \in \mathbb{R}: \; |x| \geq x \rightarrow 2|x|-2x+8>0$

$$\Rightarrow \frac{(|x|-1)(|x|-3)}{x^2-4} \le 0$$

Sea: $|x| = y$

$$\Rightarrow \frac{(y-1)(y-3)}{y^2-4} \le 0$$

$$\frac{(y-1)(y-3)}{(y-2)(y+2)} \le 0$$



$-2 < y < 1 \;\vee\; 2 < y \le 3$

$-2 < |x| \le 1 \;\vee\; 2 < |x| \le 3$

$0 \le |x| \le 1 \;\vee\; 2 < x \le 3 \;\vee\; -3 \le x < -2$

$-1 \le x \le 1 \;\vee\; 2 < x \le 3 \;\vee\; -3 \le x < -2$

C.S. = $[-3;-2[\;\cup\; [-1;1] \;\cup\;]2;3]$

10

$$|3 - |x-1|| > 1$$

$$3 - |x-1| > 1 \quad \vee \quad 3 - |x-1| < -1$$

$$2 > |x-1| \quad \vee \quad 4 < |x-1|$$

$$-2 < x-1 < 2 \quad \vee \quad 4 < x-1 \quad \vee \quad x-1 < -4$$

$$-1 < x < 3 \quad \vee \quad 5 < x \quad \vee \quad x < -3$$

$$S =]-\infty ; -3[\ \cup \ ]-1 ; 3[\ \cup \ ]5 ; +\infty[$$

$$|x-4| \leq 1 \rightarrow -1 \leq x-4 \leq 1$$

$$3 \leq x \leq 5$$

$$E = [3 ; 5]$$

$$\Rightarrow E \cap S = \emptyset$$

(12) $\sqrt{x^2-5x+4} < 7-x$

1) $x^2-5x+4 \geq 0 \quad \wedge \quad 7-x>0$

$(x-4)(x-1)\geq 0 \quad \wedge \quad x<7$

$1 \quad 4 \qquad\qquad 7$

2) $x^2-5x+4 < (7-x)^2$

$x^2-5x+4 < 49-14x+x^2$

$x<5$

5

$1 \quad 4 \quad 5 \quad 7$

$]-\infty;1] \cup [4;5[$

$\Rightarrow a+b+c=10$

(13) $\sqrt{2x+1}-\sqrt{x+8}>3$

1) $2x+1 \geq 0 \quad \wedge \quad x+8\geq 0$

$x > -\frac{1}{2} \quad \wedge \quad x \geq -8$

$-1/2 \quad \wedge \quad -8$

2) $\sqrt{2x+1} > 3+\sqrt{x+8}$

$2x+1 > 9+6\sqrt{x+8}+x+8$

$x-16 > 6\sqrt{x+8} \rightarrow x-16>0$

$x>16$

$x^2-32x+256 > 36x+288$

$x^2-68x-32>0$

$34-6\sqrt{33} \quad 34+6\sqrt{33}$

Intersectando: $x \in]34+6\sqrt{33};\infty^+[$

$\Rightarrow 34+6+33$

$=73$

13

$$\sqrt{2x+1}-\sqrt{x+8}>3$$

1) $2x+1\geq 0 \quad \wedge \quad x+8\geq 0$

$x > -\frac{1}{2} \quad \wedge \quad x \geq -8$

$-1/2 \quad \wedge \quad -8$

2) $\sqrt{2x+1} > 3+\sqrt{x+8}$

$2x+1 > 9+6\sqrt{x+8}+x+8$

$x-16 > 6\sqrt{x+8} \quad \Rightarrow \quad x-16>0$

$x>16$

$x^2-32x+256 > 36x+288$

$x^2-68x-32>0$

$34-6\sqrt{33} \qquad 34+6\sqrt{33}$

Intersectando: $x \in \left]34+6\sqrt{33}\,;\,\infty^+\right[$

$\Rightarrow 34+6+33$

$= \underline{73}$

14

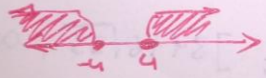
$$\frac{\sqrt{x^2-9}\cdot\sqrt[3]{x^2-16}}{\sqrt{25-x^2}} \geq 0$$

$$x^2-9\geq 0 \quad \wedge \quad 25-x^2>0$$

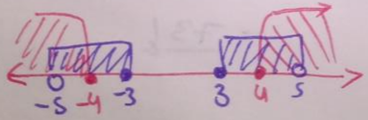
Ya que $\sqrt{x^2-9}\geq 0 \ \wedge \ \sqrt{25-x^2}>0$

$$\sqrt[3]{x^2-16}\geq 0$$

$$x^2-16\geq 0$$



Finalmente:



$$x\in\,]-5;-4] \cup [4;5[$$

15

$$\frac{\sqrt{2-|x|}\,.\,(1-x^2)}{(|x+3|+x-1)(|x|-2)} \geq 0$$

$2-|x| \geq 0 \rightarrow |x| < 2$

$\rightarrow -2 < x < 2$

$\Rightarrow \sqrt{2-|x|} > 0 \wedge (|x|-2 < 0$

$x+3 > 0 \rightarrow |x+3| = x+3$

$$\Rightarrow \frac{(1-x^2)}{(x+3+x-1)} \leq 0$$

$$\frac{1-x^2}{2x+2} \leq 0$$

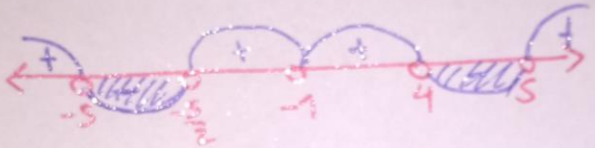
$$x \neq -1 \Rightarrow \frac{(1-x)(\cancel{1+x})}{2(\cancel{1+x})} \leq 0$$

$\rightarrow x \geq 1$

$\Rightarrow$ Intersectando: $[1;2[$

(16) $\sqrt[4]{2x+1} > 0 \to x > -\frac{1}{2}$

$$\frac{\sqrt[5]{(2x-8)}\,(x+5)^3(x-5)^3}{(x+1)^2\,(2x+5)^9} < 0$$



pero ya que $x > -\frac{1}{2}$

$\hookrightarrow x \in \,]4;5[$

(17) $A = \left\{ x \in \mathbb{R} \,/\, \frac{(x^3-1)(x^2+x+3)}{\sqrt{x-3}\cdot\sqrt[3]{x-2}} \leq 0 \right\}$

* $x-3>0 \to x>3$

$\Rightarrow x^3-1>0 \wedge x^2+x+3>0 \wedge \sqrt{x-3}>0$

$\wedge \sqrt[3]{x-2}>0 \Rightarrow A = \varnothing$

* $A^c = \mathbb{R}$

(18) $a > b > 0$

$$|x-a| + b = |x+a| - b$$

* Sea $x \geq a$

$\hookrightarrow x - a + b = x + a - b$

$\Rightarrow a = b$

$\Rightarrow \emptyset$

* $-a \leq x < a$

$\hookrightarrow \cancel{a} - x + b = x + \cancel{a} - b$

$b = x \wedge b \in [-a; a[$

$\Rightarrow \{b\}$

* $x < -a$

$\hookrightarrow a - \cancel{x} + b = -\cancel{x} - a - b$

$a = -b$

$\Rightarrow \emptyset$

$\rightarrow$ C.S. $= \{b\}$

(19)

Observamos que:

* $4 - x^2 \geq 0 \rightarrow x^2 \leq 4$

$\rightarrow -2 \leq x \leq 2$

* $x \neq 0 \wedge x \neq -1$

$\Rightarrow$ posibles elementos enteros:

$\{-2; 1; 2\}$

-2: sí cumple; 1: NO cumple; 2: sí cumple

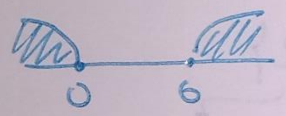
$\Rightarrow$ elementos enteros de $S \Rightarrow \{-2; 2\}$

(20)

$$\sqrt{\frac{\sqrt{x^2-6x}-\sqrt{x}}{8-x}} \geq \sqrt[3]{x-10}$$

$*\ \sqrt{x} \in \mathbb{R} \to x \geq 0$

$*\ \sqrt{x^2-6x} \in \mathbb{R} \to x^2-6x \geq 0$



$\Rightarrow x \in [6;\infty^+[\cup \{0\}$

$$\frac{\sqrt{x^2-6x}-\sqrt{x}}{8-x} \geq 0$$

Sea $x=0 \to$ cumple

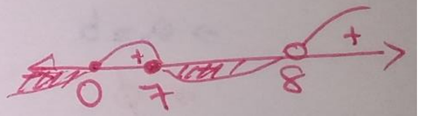
Sea $x \neq 0$

$$\frac{(\sqrt{x^2-6x}-\sqrt{x})(\sqrt{x^2-6x}+\sqrt{x})}{8-x} \geq 0$$

$$\frac{x^2-7x}{8-x} \geq 0$$

$$\frac{x^2-7x}{x-8} \leq 0$$

$$\frac{x(x-7)}{x-8} \leq 0$$



pero $x \geq 0$

$\hookrightarrow x \in [7;8[\cup \{0\}$

$x-10<0$

$$\hookrightarrow \sqrt{\frac{\sqrt{x^2-6x}-\sqrt{x}}{8-x}} \geq \sqrt[3]{x-10}$$

$\Rightarrow C.S. = [7;8[\cup \{0\}$